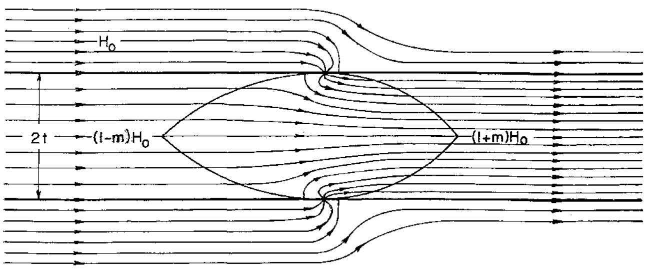

Fig. 1 The ultimate magnetic field for Alfvén number $m = \frac{1}{3}$ according to the slow-flow approximation. The vertical scale has been greatly enlarged.

no longer do magnetic lines from the outside fluid have to enter the airfoil. Secondly, features predicted by both Stewartson[2] and Sears-Resler[3] appear. The exterior flow pattern (i.e., outside the current sheets and following the magnetic lines) is a Sears-Resler potential flow over a portion of the airfoil at which there is a current sheet. However, the interior flow stemming from the remainder of the surface is a Stewartson slug flow.

The pressures between the current sheets are the same as before, but the rear pressure acts over a smaller area. Continuity of flux shows that the field under the current sheet on the body is $(1 - m)\ H_0/\alpha$, where $2\alpha t$ is the width of the airfoil at the point considered. This provides a pressure deficiency $\frac{1}{2}\mu[(1 - m)^2/\alpha^2 - 1]H_0^2$ at the body, so that the drag is now

$$\frac{1}{2}\,\mu(1 + m)^2H_0^2 \times 2\left(\frac{1 - m}{1 + m}\right) t + \frac{1}{2}\,\mu(1 - m)^2H_0^2 2t \int_{(1-m)/(1+m)}^{1} \frac{d\alpha}{\alpha^2} - \frac{1}{2}\,\mu(1 - m)^2H_0^2 \times 2t = 4m(1 - m)\mu H_0^2 t$$

and the drag coefficient is $8(1 - m)\tau/m$. The slow-flow theory overestimates the drag by a factor $1/(1 - m)$, which, in the case illustrated, is 1.5.

Details will be published shortly. Here it suffices to point out that quite novel methods are required. The perturbation within the current sheets is not small of the order of the thickness ratio, and the nonlinearity of the equations of motion must therefore be faced. This may be inferred from previous treatments of the problem, none of which attempts to do so. Fortunately the main disturbances are carried by slight modifications of Alfvén waves, which are exact solutions of the nonlinear equations.

Stewartson[4] has proposed that the following three limits be found in order to set the Sears-Resler solution[3] of the steady-state equations in its proper context: 1) vanishingly small misalignment, 2) conductivity tending to infinity, and 3) steady limit of transient solution. Our work provides the third limit and casts some doubt on whether the other two limits can be found by conventional perturbation schemes.

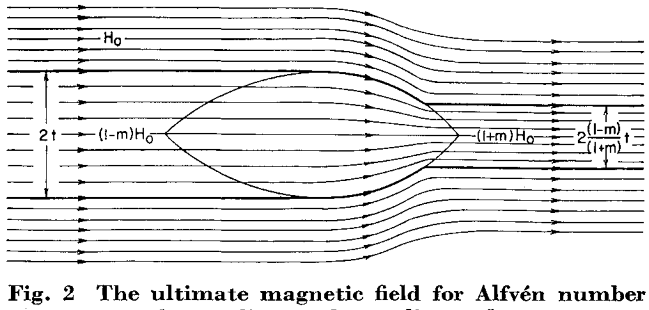

Fig. 2 The ultimate magnetic field for Alfvén number $m = \frac{1}{3}$ according to the nonlinear theory.

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One-Dimensional Flow in MHD Generators

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THE equations for the one-dimensional flow in Faraday-type MHD generators are readily solved for cases in which one of the flow variables is held constant. The solutions are well known and have been documented (for example, by Sutton[1]). But it does not appear to have been pointed out that these solutions are all particular cases of a more general solution, which follows immediately from the assumption that the ratio of loss of kinetic energy to loss of stagnation enthalpy is constant. Flows at constant velocity, constant pressure, etc., correspond to particular values of this ratio. The general solution is given here, and application of the results to closed power generation is briefly discussed.

A General Solution

The equations of momentum, energy, and state for a segmented electrode, Faraday-type generator, operating at constant load factor $K = E/vB$, are

$$\left.\begin{aligned} &\rho v\frac{dv}{dx} + \frac{dp}{dx} + \sigma v B^2(1 - K) = 0 \\ &\rho\frac{d}{dx}\left(c_p t + \frac{1}{2}v^2\right) = -\sigma BE(1 - K) = -K(1 - K)B^2\sigma v \\ &p = (\gamma - 1)c_p\rho t/\gamma \end{aligned}\right\} \quad (1)$$

where σ, E, B are the (scalar) conductivity and the electric and magnetic fields, respectively. P, p and T, t are the stagnation and static pressures and temperatures, and suffixes 1, 2 denote conditions at entry and exit from the uniform magnetic field. Other symbols have their usual significance. When these three equations are combined, one has

$$\gamma(1 - K)v\frac{dv}{dx} + c_p(K + \gamma - K\gamma)\frac{dt}{dx} + K(1 - \gamma)c_p t\frac{1}{\rho}\frac{d\rho}{dx} = 0 \quad (2)$$

In the absence of the heat losses, the output of the generator is given by the loss of total enthalpy. It is therefore convenient if H is chosen as the independent variable (particularly for application to closed cycle generator systems where optimizations are carried out in terms of total enthalpy ratios across components).

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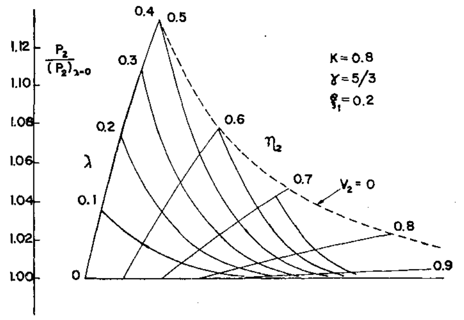

Fig. 1 Total pressure at generator exit vs η_2 and λ.

Equation (2) can be written

$$K \frac{d\xi}{d\eta} + K(\gamma - 1)(\eta - \xi) \frac{1}{\rho} \frac{d\rho}{d\eta} + (K\gamma - K - \gamma) = 0 \tag{3}$$

where

$$\xi = v^2/2H_1; \qquad \eta = H/H_1$$

are the normalized kinetic energy and total enthalpy.

Now let

$$\xi = \xi_1 + \lambda(\eta - 1) \tag{4}$$

where λ is an arbitrary constant. Upon substitution into (3) and integration, one obtains

$$\frac{\rho}{\rho_1} = \left\{\frac{(\eta - \xi_1) + \lambda(1 - \eta)}{1 - \xi_1}\right\}^{[\gamma + K(1-\gamma-\lambda)]/K(\gamma-1)(1-\lambda)} \tag{5}$$

The stagnation pressure is given by

$$\frac{P}{P_1} = \eta^{\gamma/(\gamma-1)} \left\{\frac{(\eta - \xi_1) + \lambda(1 - \eta)}{1 - \xi_1}\right\}^{\gamma(1-K)/K(\gamma-1)(1-\lambda)} \tag{6}$$

which reduces to the isentropic relationship for $K = 1$.

Apart from an expression for the duct station x, the flow is now determined. The velocity and density are given by Eqs. (4) and (5), the static temperature is $H_1(\eta - \xi)/c_p$, and the static pressure is given by the equation of state. The following cases are included in Eq. (4):

Constant Velocity

$$\lambda = 0$$

Constant Mach Number

$$\lambda = \xi_1 = 1 - 2/\{2 + (\gamma - 1)M_1^2\}$$

Constant Temperature

$$\lambda = 1$$

Constant Pressure

$$\lambda = 1/K$$

Constant Density

$$\lambda = (\gamma - \gamma K + K)/K$$

For the constant temperature case, in the limit $\lambda \to 1$, the expressions (5) and (6) become

$$\frac{P}{P_1} = \eta^{\gamma/(\gamma-1)} \frac{\rho}{\rho_1} = \eta^{\gamma/(\gamma-1)} \exp\left\{\frac{-(1 - K)(1 - \eta)\gamma}{K(\gamma - 1)(1 - \xi_1)}\right\} \tag{7}$$

Solutions previously given for the other four cases are recovered when the preceding values of λ are inserted into Eqs. (5) and (6). The presence of the parameter λ therefore introduces a degree of generality which should be useful in the design of MHD ducts and in the evaluation of experiments.

When $\lambda < 1$, power is extracted at the expense of both kinetic energy and static enthalpy. For $\lambda > 1$, rapid diffusion of the fluid produces both power output and an increase in static temperature.

Since t and v must remain positive,

$$\eta - \xi_1 + \lambda(1 - \eta) > 0$$

$$\xi_1 - \lambda(1 - \eta) > 0$$

The second of these inequalities illustrates rather simply that, for operation at high values of λ, such as constant temperature or pressure, an appreciable fraction of the total enthalpy can only be extracted if the inlet Mach number is high.

Generator Length

Swift-Hook and Wright,[2] in their analysis of the constant Mach number MHD generator, have suggested that the conductivity be given by $\sigma \sim t^y p^{-z}$, where y and z are constants chosen so as to fit theoretically or experimentally obtained values of the conductivity. Apart from its convenience, such a power law is certainly quite adequate, whatever the technique by which the conductivity is produced. (It may be noted that the temperature range for practical operation narrows as the temperature dependence of conductivity becomes stronger.) In the present notation,

$$\sigma = \sigma_1(t/t_1)^y(p/p_1)^{-z} = \sigma_1 \left\{\frac{(\eta - \xi_1) + \lambda(1 - \eta)}{1 - \xi_1}\right\}^{y-z} (\rho/\rho_1)^{-z} \tag{8}$$

Upon substitution of (5) and (8) into (1), one has, unless $\lambda = 1$,

$$dx/d\eta = -(H_1/2)^{1/2}\rho_1 g(\eta;\ \lambda)/K(1 - K)\sigma_1 B^2 \tag{9}$$

where

$$g(\eta;\ \lambda) = \left\{\frac{(\eta - \xi_1) + \lambda(1 - \eta)}{1 - \xi_1}\right\}^{\beta} \{\xi_1 - \lambda(1 - \eta)\}^{-1/2}$$

$$\beta = \frac{\gamma(1 + z)(1 - K\lambda) - K(\gamma - 1)(1 + y)(1 - \lambda)}{K(\gamma - 1)(1 - \lambda)}$$

The duct station x and the duct length can be expressed in closed form for constant velocity [$\lambda = 0$ in Eq. (9)] and for constant Mach number ($\lambda = \xi_1$). Otherwise, the integration can be carried out only for $\beta = \pm n/2$, where n is an integer.

A general expression for duct station, even if obtainable, would probably not be of great interest. What is of interest, particularly for closed loop generating systems, is the variation of stagnation pressure drop and generator length with λ. From Eq. (6), one has

$$\frac{K(\gamma - 1)(1 - \lambda)^2}{(1 - K)\gamma P} \frac{\partial P}{\partial \lambda} = f(\eta;\ \lambda)$$

where

$$(\eta;\ \lambda) = \frac{(1 - \eta)(1 - \lambda)}{(\eta - \xi_1) + \lambda(1 - \eta)} + \ln\left\{\frac{(\eta - \xi_1) + \lambda(1 - \eta)}{1 - \xi_1}\right\} = \frac{(1 - \eta)(1 - \lambda)}{(1 - \xi_1)} \frac{t_1}{t} + \ln\left(\frac{t}{t_1}\right)$$

It can be shown easily that $f(\eta;\ \lambda) > 0$ for all $\eta(<1)$ and $\lambda(>0)$. One need only note that $t/t_1 < 1$ for $\lambda < 1$, and $t/t_1 > 1$ for $\lambda > 1$. Therefore, $P(\eta_2;\ \lambda)$ increases monotonically with λ. The variation of $P_2/(P_2)_{\lambda=0}$ with η_2 and λ is illustrated in Fig. 1 for typical values of γ, K, and ξ_1.

The variation of duct length with λ is more complicated. If $L(\eta_2;\ \lambda)$ is the generator length for an output $H_1(1-\eta_2)$ per unit of mass flow, then from (9)

$$\frac{\partial L}{\partial \lambda} = \frac{(H_1/2)^{1/2}\rho_1}{K(1-K)B^2\sigma_1}\int_{\eta=\eta_2}^{1} g\,\frac{\partial}{\partial\lambda}(\ln g)\,d\eta$$

$$\frac{(1+z)(1-K)}{K(1-\lambda)^2(1-\eta)}\left(\frac{\partial\beta}{\partial\lambda}\right)^{-1}\frac{\partial}{\partial\lambda}(\ln g) = \frac{\gamma-1}{2\gamma\{\xi_1-\lambda(1-\eta)\}} + \frac{1}{\gamma}\left\{\frac{\gamma(1+z)-(\gamma-1)(1+y)}{(\eta-\xi_1)+\lambda(1-\eta)}\right\} + \frac{(1+z)(1-K)}{K(1-\lambda)^2(1-\eta)}f(\eta;\ \lambda) \quad (10)$$

Now $g>0$; $f>0$; $\partial\beta/\partial\lambda>0$ for all η_1, λ, so that $\mathrm{sgn}(\partial L/\partial\lambda)$ is given by the sign of the right side of Eq. (10) in which only the second term can be negative. Note that $\partial(\ln g)/\partial\lambda = 0$ for $\eta = 1$. Clearly, for y less than, or slightly greater than, $(z\gamma+1)/(\gamma-1)$, the generator length increases monotonically with increasing λ for all permissible values of the parameters. For large values of y, the second term of Eq. (9) is negative and becomes the dominant term as η decreases from 1 to η_2.

Closed Cycle Power Generation

The conclusions reached previously may be stated as follows: The value of λ determines what fractions of the total power are extracted at the expense of kinetic energy and of static enthalpy. For $\lambda = 0$, energy is extracted from the static enthalpy only; when $\lambda = 1$ power is taken from the kinetic energy only. For a given total power per unit mass flow, the loss of stagnation pressure decreases as λ increases. When the temperature dependence of conductivity is weak, the generator length increases as λ increases. For strong temperature dependence, this effect is reversed.

These results are, of course, not surprising, and could be deduced from physical considerations or from an examination of the initial equations. What they are intended to illustrate here is the usefulness of the parameter λ in defining generator operation. There is no reason why one should be restricted to constant velocity, temperature, etc., as seems to have been the case hitherto. Analyses of closed cycle MHD-turbine generating plants, such as those of Gunson et al.,[3] or of Sodha and Bendor,[4] are usually carried out for constant velocity or constant Mach number when, in fact, the selection of a somewhat higher value of λ would have led to a reduction in both the generator length and the pressure drop.

The generator dimensions enter the efficiency estimation of a closed cycle system through the heat lost from the duct and the power required to produce the magnetic field. These are given by the integral over the generator length of the product of the temperature difference between gas and duct walls and some transverse dimension of the duct. Expression could be written down for the heat loss (as in Sodha and Bendor[4] for constant M only), and their variation with λ examined. It can also be shown that the duct dimensions at exit increase with increasing λ for given entry conditions and η_2. Thus, for weak temperature dependence of conductivity, the heat loss increases more rapidly with λ than the generator length. For a rapidly varying conductivity, one may expect the positive variation of closed cycle efficiency with λ, resulting in this case both from a reduction in pressure loss and in generator length, to be reduced, and for high values of λ, even reversed.

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Radial Lag of Solid Particles in Delaval Nozzles

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RECENTLY a light scattering technique has been used to study particle trajectories in nozzle plume gas-particle flows. In this note, experimental measurements of the radial spreading of the particle cloud are compared with results calculated by a computer program developed to determine solid particle velocity and thermal lags in rocket nozzles.[1] The particle spreading at the nozzle exit is compared with theory for both large (glass beads) and small (alumina) particles. The observed particle boundary was found to compare well with the theoretical limiting particle streamline‡ for the small size particles from each powder size distribution.

The experimental facility consisted of a 3800 ft^3 blow-down vacuum tank through which a mixture of nitrogen gas and solid particles could be fed and studied with appropriate instrumentation. The mixing of the particles and the gas was accomplished by forcing the particles into the gas stream with a piston driven by gas pressure. In addition to light scattering photography, instrumentation included measurements of stagnation, exit plane, and tank pressures and gas and particle flow rates. A conical nozzle was used

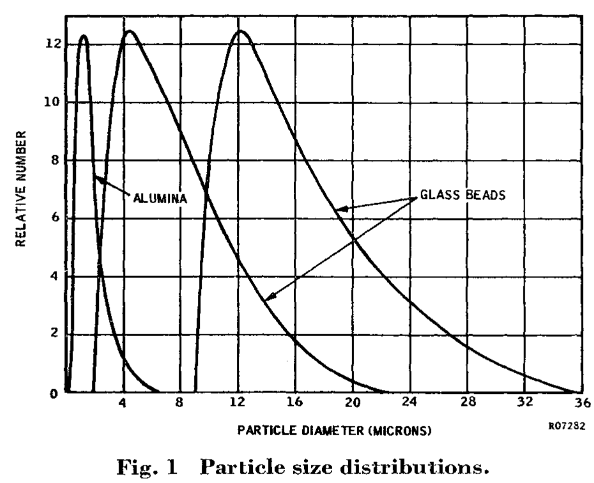

Fig. 1 Particle size distributions.

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‡ The "limiting streamline" for a given size particle is defined as that particle trajectory that passes closest to the nozzle wall without impingement.